

AMERICAN UNIVERSITY OF BEIRUT  
STATISTICS 238, Final Exam

Jan 23, 2003

Time = 1 Hour and 30 Minutes



You are allowed to use a formula sheet.

1. Suppose we have two boxes and  $2d$  balls, of which  $d$  are black and  $d$  are red. Initially,  $d$  balls are placed in box 1, and the remainder of the balls are placed in box 2. At each trial a ball is chosen at random from each of the boxes, and the two balls are put back in the opposite boxes. Let  $X_0$  denote the number of black balls initially in box 1 and, for  $n \geq 1$ , let  $X_n$  denote the number of black balls in box 1 after the  $n$ th trial. Find the transition matrix of the Markov chain  $X_n$ .
2. Let  $X_n, n \geq 0$ , be two state Markov chain.
  - (a) Find  $P_0(T_0 = n)$ .
  - (b) Find  $P_1(T_1 = n)$ .
3. Consider the following transition matrix:

$$P = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1/4 & 1/4 & 0 & 1/2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 & 2/3 \end{pmatrix}$$

Classify the states of the above chain.

4. Suppose that all car owners fill up when their tanks are exactly half full. At the present time, an average of 7.5 customers per hour arrive at a single pump gas station. It takes an average of 4 minutes to service a car. Assume that interarrival times and service times are both exponentially distributed.
  - (a) Compute  $L$  and  $W$ .
  - (b) Suppose that a gas shortage occurs and panic buying takes place. To model this phenomenon, suppose that all car owners now purchase gas when their tanks are exactly three quarters full. Since each car owner is now putting less gas into the tank during each visit to the station, we assume the average service time has been reduced to  $3\frac{1}{3}$  minutes. How has panic buying affected  $L$  and  $W$ ?
5. If two identical M/M/1 queuing systems were to be used instead an M/M/2 queuing system with arrival rate  $\lambda$  and service rate  $\mu$  where switching between lines wouldn't be allowed. What changes would be incurred to the proportion of time that the system stays idle?